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PREDICTIVE MODELING FOR SHARE CLOSING PRICES THROUGH HIDDEN MARKOV MODELS WITH A SPECIAL REFERENCE TO THE NATIONAL STOCK EXCHANGE

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Abstract

Predicting share prices is crucial in financial markets for traders and portfolio managers. This study employs hidden Markov modeling, focusing on three key parameters: initial probability vector (IPV), transition probability matrix (TPM), and emission/observed

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probability matrix (EPM/OPM). TPM and EPM are derived by considering both hidden and emission states. Probability distributions are formulated for increment, remain same, and decrement states. The model's behaviour is explored through statistical measures and Pearson's coefficients. This model aids in estimating stock price movements, and long-term and short-term returns, and can be compared with the capital asset pricing model (CAPM). Numerical illustrations are used for clarity, and model goodness of fit is assessed with the chi-square test. Developing user-friendly digital interfaces can enhance traders' understanding of Wipro's stock market behaviour in the Indian context.

1. Introduction

Modeling is a core activity of constructing the structural connectivity between the study variables. The objective of modeling is to explore the influence of the factor variables on the response variables. Markov processes deal with the stochastic processes of memoryless property in which the happening of the current event is influenced by the immediate previous event only, not by the remaining previous events. Markov modeling deals with studying the transit behaviour of the states in any time series data. The transition probability matrix (TPM) provides all the possible chances of different combinations between the previous and the current trial's results (states). Hidden Markov modeling is an activity of building a relational processing structure between the observed/emission and the hidden states.

The usage of stochastic models for predicting the dynamics of the stock market is an established research methodology. Much research literature study is reported on measuring the stock values. The models namely Brownian motion processes, Weiner processes, Black-Scholes model, birth-death processes, investment and liquidation processes, Markov process, hidden Markov model (HMMs) etc., have been considered for predicting the different parameters of the stock market.

Making use of stochastic calculus by means of differential equations is the most commonness among all the above mentioned models. The above models are successful in predicting the market behaviour with desired vital parameters such as capital asset pricing, returns on income (ROI), risk adjusted return on capital (RAROC), return on risk adjusted capital (RORAC), risk adjusted return on risk adjusted capital (RARORAC), etc. Further, these models have made the conventional methodologies in making use of probability generating functions (PGFs) for estimating the desired parameters.

Black and Scholes [3] studied the dynamics of prices in derivative securities. Black-Scholes formula for the price of a European option is one of the first fundamental results in this direction.

Bachelier [1] formulated a theory to analyze the movement of asset prices under risk, employing Brownian motion and assuming that the increments in stock prices are independent. This theory was specifically tailored for application in the Paris stock market. Neftci [2] developed Weiner Kolmogorov's prediction theory for forecasting the financial market. Hassan et al. [5] proposed and implemented a fusion model by combining the HMM, artificial neural networks (ANNs) and genetic algorithms (GAs) to forecast financial market behaviour. Rao and Hong [6] used HMM and support vector machines (SVMs) to forecast the stock price and its movements. Padi and Gudala [4] developed the bivariate Poisson model with the incomings, outgoings, and mutual transfers of investments between and within the portfolios using stochastic differential equations and the notions of bivariate linear birth, death, and migration processes. Adesokan et al. [7] analyzed the Nigerian stock market expected returns using the Markov chain and capital asset price model (CAPM). Su and Yi [8] proposed the HMM for the efficient prediction of stock price in the financial market. Dar et al. [9] used the Markov chain model for the analysis of the stock movement and forecasting the share prices of TCS Ltd. Padi et al. [10] applied the discrete-time Markov chain to analyze the behaviour of share prices with reference to SBI. Dar et al. [11] used HMM for a proper understanding of the financial variables in the stock market.

Limited evidence exists for deriving probability mass functions of states with varying sequence lengths. Predictive modeling, particularly for Markov processes, lacks attention in parameter estimation. Exploring transit state probability distributions can provide better information. Parametric estimation within the Markov model and extending to probability distributions requires further attention from probability researchers.

This study addresses the gap by emphasizing the importance of hidden Markov modeling to establish relationships between emission and hidden variables. It focuses on formulating PMFs for discrete hidden Markov processes, particularly the length of state sequence distributions. The study derives explicit mathematical relations for various statistical measures based on these probability distributions.

In this study, hidden Markov models (HMMs) are used to analyze NSE closing prices of Wipro's shares, with three hidden states: gain, normal, and fall, corresponding to observed states of increment, remain same, and decrement. The key parameters are the initial probability vector (IPV), transition probability matrix (TPM), and observed probability matrix (OPM). The study aims to (i) formulate probability distribution models for increment, remain same, and decrement states, (ii) derive mathematical relations for statistical measures and Pearson's coefficients, (iii) analyze real-time NSE and Wipro's closing prices to find HMM parameters, (iv) understand market behaviour through probability distributions and statistical measures.

2. Stochastic Model

This model aims to derive the discrete probability distributions of the number of different emission states. Here, we have considered three transition emission states, namely (i) increment state, (ii) remain same state, and (iii) decrement state.

2.1. Notations and terminology

π_i : Initial probability for ith hidden state, $\pi_i \geq 0;$ for all $i = 1, 2, 3.$

X_n : Resulting value of hidden states at nth trail, $n = 1, 2, 3, \ldots$.

Y_m : Resulting value of emission/observed state with the influence of the related hidden states at mth trail, $m = 1, 2, 3, \ldots$.

a_{kl} : The transition probability within hidden states; $P\{X_n = l / X_{n-1} = k\} \geq 0;\ 0 \leq a_{kl} \leq 1$.

b_{kl} : The emission/observed probability between hidden and emission state; $P\{Y_m = l / X_{m-1} = k\} \geq 0;\ 0 \leq b_{kl} \leq 1$.

x_t : Share price of the NSE at tth day.

Δx_t : $x_t - x_{t-1}$; the difference between the current day (t) share value and previous day $(t-1)$ share value NSE.

dx_t : The dividend at time 't'; $dx_t = \dfrac{\Delta x_t}{x_{t-1}}$.

n : The number of observations in NSE considered under study.

μ_x : Mean dividend value of the share; i.e., $\mu_x = \dfrac{1}{n}\sum_{t=1}^{n} dx_t$.

σ_x^2 : Variance of the dividend value of the share, i.e., $\sigma_x^2 = \dfrac{1}{n-1}\sum_{t=1}^{n}(dx_t - \mu_x)^2$.

y_t : Share price of the Wipro's Ltd. at tth day.

Δy_t : $y_t - y_{t-1}$; the difference between the current day (t) share value and previous day $(t-1)$ share value in Wipro Ltd.

dy_t : The dividend of Wipro at time 't'; $dy_t = \dfrac{\Delta y_t}{y_{t-1}}$.

m : The number of observations in Wipro considered under study.

μ_y : Mean dividend value of the share of Wipro, i.e., $\mu_y = \frac{1}{m}\sum_{t=1}^{m} dy_t$.

σ_y^2 : Variance of the dividend value of the share of Wipro, i.e., $\sigma_y^2 = \frac{1}{m-1}\sum_{t=1}^{m}(dy_t - \mu_y)^2$.

2.2. Schematic diagram for three states HMM of one day length

The schematic diagram of HMM for increment state occurring in one day length is given below:

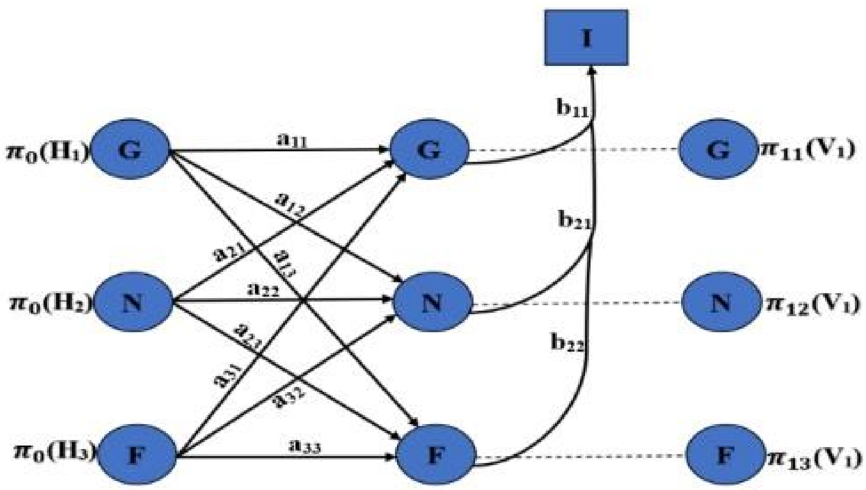

Figure 1. HMM for increment state occurs is one day length.

A similar schematic diagram follows for

(i) HMM for remain same (R) state of length one.

(ii) HMM for decrement (D) state of length one.

2.3. Schematic diagram for three states HMM of two days length

The schematic diagram of HMM for increment state occurring in a length of sequence two is given below:

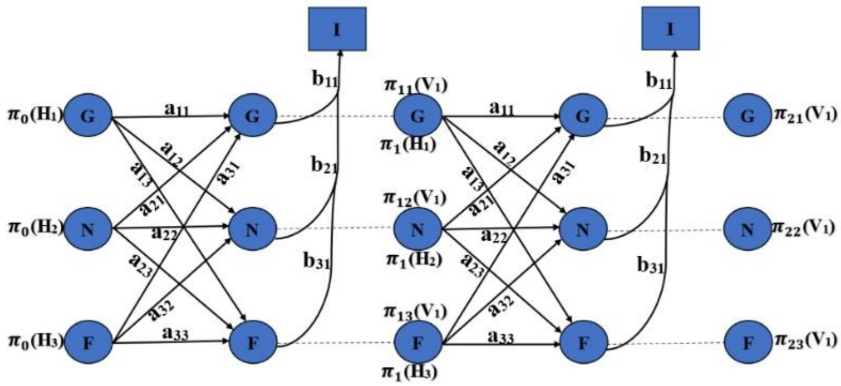

Figure 2. HMM for two days increment (I) state occurs in a length of sequence two.

Similar schematic diagrams can be obtained for (a) HMM for first day is on increment (I) state second day is on remain same (R) state in a length of sequence two, (b) HMM for first day is on increment (I) state, second day is on decrement (D) state in a length of sequence two, (c) HMM for first day is on remain same (R) state, second day is on increment (I) state in a length of sequence two, (d) HMM for both days remain same (R) state in a length of sequence two, (e) HMM for first day is on remain same (R) state, second day is on decrement (D) state in a length of sequence two, (f) HMM for first day is on decrement (D) state, second day is on increment (I) state in a length of sequence two, (g) HMM for first day is on decrement (D) state, second day is on remain same (R) state in a length of sequence two, (h) HMM for both days decrement (D) state in a length of sequence two.

3. Probability Distribution for One Day Sequence

Let $X(\omega_{m1}) = m$ be the random variable, which denotes the happening of the state. ω_{m1} is indicated with three different states, namely increment state, remain same state, and decrement state, i.e., $\omega_{m1} = I$ or R or D. Let 'm' be the number of times the event happening in that state, $m = 0, 1,$ where '0' represents non-happening of the state and '1' represents the happening of the state.

We use the following notations in Subsection 3.1:

$$P_{x1}(V_1) = \sum_{i=1}^{3}\sum_{j=1}^{3}[\pi_0(H_i)a_{ij}b_{j1}];$$

$$P_{x1}(V_2) = \sum_{i=1}^{3}\sum_{j=1}^{3}[\pi_0(H_i)a_{ij}b_{j2}];$$

$$P_{x1}(V_3) = \sum_{i=1}^{3}\sum_{j=1}^{3}[\pi_0(H_i)a_{ij}b_{j3}].$$

3.1. Probability mass function for increment state occurs in one day length

The probability mass function for increment state occurring in one day length is as follows:

$$P(X(\omega_{m1}) = x) = \begin{cases} \sum_{l=2}^{3} P_{x1}(V_l); & x = 0, \\ P_{x1}(V_1); & x = 1. \end{cases} \tag{3.1}$$

3.1.1. Statistical measures for increment state occurs in one day length

3.1.1.1. Average number of times happening of the increment state is

$$\mu_1^1 = P_{x1}(V_1). \tag{3.2}$$

3.1.1.2. Standard deviation for increment state is

$$\sigma = \left[{\mu_1^1}^2 \sum_{l=2}^{3} P_{x1}(V_l) + (1-\mu_1^1)^2 P_{x1}(V_1)\right]^{1/2}. \tag{3.3}$$

3.1.1.3. The third central moment for increment state is

$$\mu_3 = \left[-{\mu_1^1}^3 \sum_{l=2}^{3} P_{x1}(V_l) + (1-\mu_1^1)^3 P_{x1}(V_1)\right]. \tag{3.4}$$

3.1.1.4. Karl-Pearson's coefficient of skewness for increment state is

$$\beta_1 = \left[-\mu_1^{1^3}\sum_{l=2}^{3} P_{x1}(V_l) + (1-\mu_1^1)^3 P_{x1}(V_1)\right]^2$$

$$\times\left[\mu_1^{1^2}\sum_{l=2}^{3} P_{x1}(V_l) + (1-\mu_1^1)^2 P_{x1}(V_1)\right]^{-3}. \qquad (3.5)$$

3.1.1.5. Coefficient of kurtosis for increment state is

$$\beta_2 = \left[\mu_1^{1^4}\sum_{l=2}^{3} P_{x1}(V_l) + (1-\mu_1^1)^4 P_{x1}(V_1)\right]$$

$$\times\left[\mu_1^{1^2}\sum_{l=2}^{3} P_{x1}(V_l) + (1-\mu_1^1)^2 P_{x1}(V_1)\right]^{-2}. \qquad (3.6)$$

Similar lines follow for

(i) HMM for remain same state of length one.

(ii) HMM for decrement state of length one.

4. Probability Distribution for Two Days Sequence

Let $X(\omega_{m2}) = m$ be the random variable which denotes the happening of the state. ω_{m2} indicates three different states, namely increment state, remain same state, and decrement state, i.e., $\omega_{m2} = I$ or R or D. 'm' be the number of times happening of the event in that particular state, $m = 0, 1, 2$, where '0' represents non-happening of the state in two days length, '1' represents happening of the state once in two days length, and 2 represents the happening of the state twice in two days length.

Below notations are going to use in Subsection 4.1:

$$P_{x1}(V_1) = \sum_{i=1}^{3}\sum_{j=1}^{3}[\pi_0(H_i)a_{ij}b_{j1}];$$

$$P_{x1}(V_2) = \sum_{i=1}^{3}\sum_{j=1}^{3}[\pi_0(H_i)a_{ij}b_{j2}];$$

$$P_{x1}(V_3) = \sum_{i=1}^{3}\sum_{j=1}^{3}[\pi_0(H_i)a_{ij}b_{j3}];$$

$$P_{x2}(V_1) = \sum_{i=1}^{3}\sum_{j=1}^{3}[\pi_1(H_i)a_{ij}b_{j1}];$$

$$P_{x2}(V_2) = \sum_{i=1}^{3}\sum_{j=1}^{3}[\pi_1(H_i)a_{ij}b_{j2}];$$

$$P_{x2}(V_3) = \sum_{i=1}^{3}\sum_{j=1}^{3}[\pi_1(H_i)a_{ij}b_{j3}].$$

4.1. Probability mass function for increment state occurs in two days length

The probability mass function for increment state occurring in two days length is as follows:

$$P(X(\omega_{m2}) = x) = \begin{cases} \prod_{l=1}^{2}\left(\sum_{m=2}^{3} P_{xl}(V_m)\right); & x = 0, \\ P_{x2}(V_1)\sum_{m=2}^{3} P_{x1}(V_m) + P_{x1}(V_1)\sum_{m=2}^{3} P_{x2}(V_m); & x = 1, \\ \prod_{m=1}^{2} P_{xm}(V_1); & x = 2. \end{cases} \quad (4.1)$$

4.1.1. Statistical measures for increment state occurs in two days length

4.1.1.1. Average number of times happening of increment state is

$$\mu_1^1 = \left[P_{x2}(V_1)\sum_{m=2}^{3} P_{x1}(V_m) + P_{x1}(V_1)\sum_{m=2}^{3} P_{x2}(V_m)\right] + 2\left(\prod_{m=1}^{2} P_{xm}(V_1)\right). \quad (4.2)$$

4.1.1.2. Standard deviation for increment state is

$$\sigma = \left\{ \mu_1^{1^2} \left[\prod_{l=1}^{2} \left(\sum_{m=2}^{3} P_{xl}(V_m) \right) \right] + (1-\mu_1^1)^2 \left[P_{x2}(V_1) \sum_{m=2}^{3} P_{x1}(V_m) + P_{x1}(V_1) \sum_{m=2}^{3} P_{x2}(V_m) \right] + (2-\mu_1^1)^2 \left(\prod_{m=1}^{2} P_{xm}(V_1) \right) \right\}^{1/2}. \quad (4.3)$$

4.1.1.3. The third central moment for increment state is

$$\mu_3 = -\mu_1^{1^3} \left[\prod_{l=1}^{2} \left(\sum_{m=2}^{3} P_{xl}(V_m) \right) \right] + (1-\mu_1^1)^3 \left[P_{x2}(V_1) \sum_{m=2}^{3} P_{x1}(V_m) + P_{x1}(V_1) \sum_{m=2}^{3} P_{x2}(V_m) \right] + (2-\mu_1^1)^3 \left(\prod_{m=1}^{2} P_{xm}(V_1) \right). \quad (4.4)$$

4.1.1.4. Karl-Pearson's coefficient of skewness for increment state is

$$\beta_1 = \left\{ -\mu_1^{1^3} \left[\prod_{l=1}^{2} \left(\sum_{m=2}^{3} P_{xl}(V_m) \right) \right] + (1-\mu_1^1)^3 \left[P_{x2}(V_1) \sum_{m=2}^{3} P_{x1}(V_m) + P_{x1}(V_1) \sum_{m=2}^{3} P_{x2}(V_m) \right] + (2-\mu_1^1)^3 \left(\prod_{m=1}^{2} P_{xm}(V_1) \right) \right\}^2 \left\{ \mu_1^{1^2} \left[\prod_{l=1}^{2} \left(\sum_{m=2}^{3} P_{xl}(V_m) \right) \right] \right.$$

$$+ (1-\mu_1^1)^2\left[P_{x2}(V_1)\sum_{m=2}^{3}P_{x1}(V_m) + P_{x1}(V_1)\sum_{m=2}^{3}P_{x2}(V_m)\right]$$

$$+ (2-\mu_1^1)^2\left(\prod_{m=1}^{2}P_{xm}(V_1)\right)\Bigg\}^{-3}. \tag{4.5}$$

4.1.1.5. Coefficient of Kurtosis for increment state is

$$\beta_2 = \Bigg\{\mu_1^{1^4}\left[\prod_{l=1}^{2}\left(\sum_{m=2}^{3}P_{xl}(V_m)\right)\right]$$

$$+ (1-\mu_1^1)^4\left[P_{x2}(V_1)\sum_{m=2}^{3}P_{x1}(V_m) + P_{x1}(V_1)\sum_{m=2}^{3}P_{x2}(V_m)\right]$$

$$+ (2-\mu_1^1)^4\left(\prod_{m=1}^{2}P_{xm}(V_1)\right)\Bigg\}\Bigg\{\mu_1^{1^2}\left[\prod_{l=1}^{2}\left(\sum_{m=2}^{3}P_{xl}(V_m)\right)\right]$$

$$+ (1-\mu_1^1)^2\left[P_{x2}(V_1)\sum_{m=2}^{3}P_{x1}(V_m) + P_{x1}(V_1)\sum_{m=2}^{3}P_{x2}(V_m)\right]$$

$$+ (2-\mu_1^1)^2\left(\prod_{m=1}^{2}P_{xm}(V_1)\right)\Bigg\}^{-2}. \tag{4.6}$$

Similar lines follow for

(i) HMM for remain same state of length two.

(ii) HMM for decrement state of length two.

5. Estimating Expected Returns

Expected returns are computed using the below formulas.

5.1. Expected long-run returns

The formula for computing long-run is

$$\mu_R = B^n \mu_k. \tag{5.1}$$

Here 'μ_R' is the long-run returns, 'B^n' is the steady state probability, and 'μ_k' is the mean return of state 'k'.

5.1.1. Expected short-run returns

The formula for computing short-run is

$$\mu_r = B^n \mu_k. \tag{5.2}$$

Here 'μ_r' is the short-run return, 'B^n' is the limiting probability, and 'μ_k' is the mean return of state 'k'.

5.2. Prediction of share price

The formula for predicting the share price is given below:

$$U_{t+1} = [U_t \times \Delta_t] + U_t. \tag{5.3}$$

Here, $U_{t+1} =$ predicted value of the current day share price, $U_t =$ value of the previous day share price, $\Delta_t =$ difference between value of previous day share price and current day share price, i.e., $\Delta_t = U_{t+1} - U_t$.

5.3. Capital asset price model (CAPM)

CAPM is a straightforward and effective tool used by investors to determine the systematic risk and expected returns of asset, money, share or other securities. In essence, the model explains how much compensation should be anticipated for taking a risk.

The formula for computing the CAPM is

$$E_r = r_f + \beta(r_m - r_f). \tag{5.4}$$

Here, 'E_r' is the expected return of the share, 'r_f' is the risk-free rate, 'β' is the risk measure, and 'r_m' is the expected market return.

The high value of ‘β’ is indicating the risk of the stock. Hence, the investor expects more returns. If the value of ‘β’ is more than ‘1’, then it is an indication to the investor that his stock is in more risk than market risk. Hence, the investor can expect high returns. If the value of ‘β’ is less than ‘1’, then it indicates that the investors stock risk is less compared to the market risk. Hence, the investor can expect low returns. The risk is directly proportional to the expected returns, i.e., $\text{Risk}(\beta) \propto \text{Returns}(E_r)$.

The market risk premium $(r_m - r_f)$ is a term which describes the relation between expected market return and risk-free rate.

6. Sensitivity Analyses

In order to make use of the developed HMM, a real-time data on closing prices of NSE and Wipro Company is considered. Data on closing prices of the stock market about 248 observations are collected for the period from 16th March 2021 to 15th March 2022 from the website http://in.finance.yahoo.com. Classification of the transient state in NSE has been carried out by using dividend (dx_t). State-G (hidden state-1) is considered if $dx_t \geq \mu_x + \frac{\sigma_x}{\sqrt{n}}$, State-$N$ (hidden state-2) is considered if $\mu_x - \frac{\sigma_x}{\sqrt{n}} < dx_t < \mu_x + \frac{\sigma_x}{\sqrt{n}}$, and State-$F$ (hidden state-3) is considered if $dx_t \leq \mu_x - \frac{\sigma_x}{\sqrt{n}}$. Wipro’s dividend (dy_t) is classified as the emission states by assuming State-I (emission state-1) is considered when $dy_t \geq \mu_y + \frac{\sigma_y}{\sqrt{m}}$, State-$R$ (emission state-2) is considered when $\mu_y - \frac{\sigma_y}{\sqrt{m}} < dy_t < \mu_y + \frac{\sigma_y}{\sqrt{m}}$, and State-$D$ (emission state-3) is considered when $dy_t \leq \mu_y - \frac{\sigma_y}{\sqrt{m}}$.

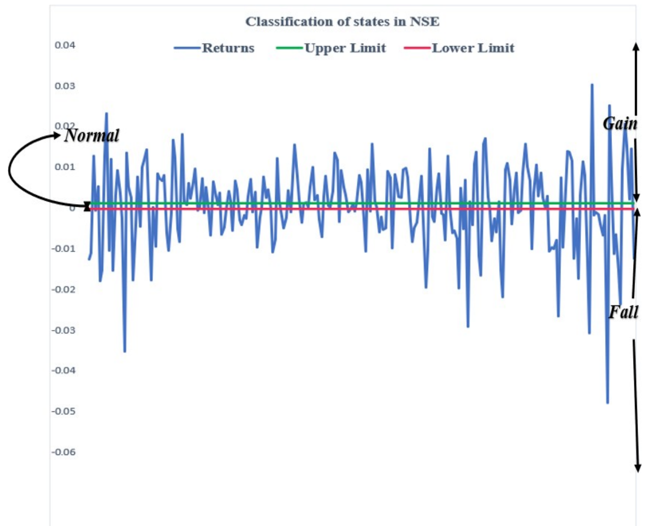

Figure 3. NSE's classification of states.

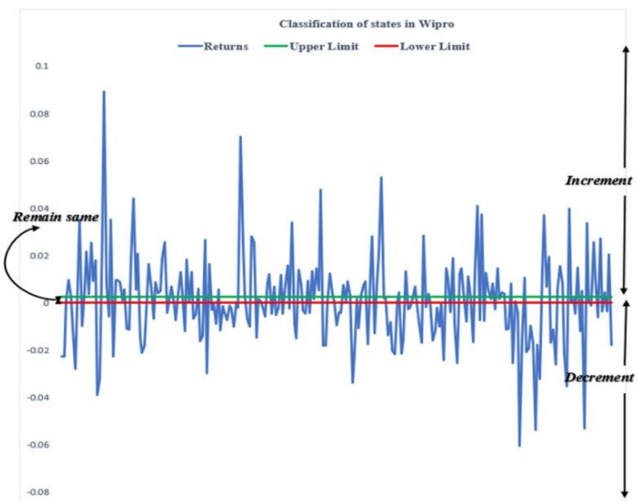

Figure 4. Wipro's classification of states.

Figures 3 and 4 exhibit the fluctuations of share values. They also exhibit the classifications of states of NSE and Wipro shares. It gives the indication to the investors and portfolio managers regarding the optimal time of selling and buying of the Wipro's share. A detailed numerical illustration is provided in the next section.

The HMM parameters in $\lambda = (A, B, \pi)$ can be explored by the computing TPM, EPM/OPM, and IPV, through MS Excel and R Programming. Separate probability distributions are obtained for increment state, remain same state, and decrement state. Statistical characteristics/measures like mean, standard deviation, third central moments, Pearson's coefficient of skewness and kurtosis etc. are obtained through derived mathematical expressions using the mentioned software. The capital asset price model (CAPM), and estimated expected returns are calculated using the literature. The goodness of fit of the developed model is tested by using chi-square test.

6.1. Numerical illustration and results discussions

6.1.1. Parameters of the HMM

The explored parameters for HMM, i.e., $\lambda = (A, B, \pi)$ is as follows:

$$A = \begin{matrix} & G & N & F \\ G & 0.552846 & 0.065041 & 0.382114 \\ N & 0.571429 & 0.071429 & 0.357143 \\ F & 0.431193 & 0.045872 & 0.522936 \end{matrix},$$

$$B = \begin{matrix} & I & R & D \\ G & 0.658537 & 0.113821 & 0.227642 \\ N & 0.214286 & 0.071429 & 0.714286 \\ F & 0.227273 & 0.090909 & 0.681818 \end{matrix},$$

$$\pi = \begin{matrix} G & N & F \\ (0.497976 & 0.05668 & 0.445344). \end{matrix}$$

It shows that the NSE of the gain state has the highest likelihood with 49.8%, fall state has second highest likelihood with 44.53%, and normal state has least likelihood with 5.67%. On average, it reveals that the state of gain state is having more advantages than remaining two states. It indicates that the market may boom in upcoming days.

6.1.2. Stationary matrix for TPM and EPM/OPM

The stationarity will be achieved for TPM at 9th consecutive day and for EPM/OPM at 20th consecutive day. This is given below:

$$A^9 = \begin{array}{c} \\ G \\ N \\ F \end{array}\begin{array}{c} \begin{array}{ccc} G & N & F \end{array} \\ \begin{bmatrix} 0.5 & 0.056911 & 0.443089 \\ 0.5 & 0.056911 & 0.443089 \\ 0.5 & 0.056911 & 0.443089 \end{bmatrix} \end{array},$$

$$B^{20} = \begin{array}{c} \\ G \\ N \\ F \end{array}\begin{array}{c} \begin{array}{ccc} I & R & D \end{array} \\ \begin{bmatrix} 0.39737 & 0.098103 & 0.504527 \\ 0.39737 & 0.098103 & 0.504527 \\ 0.39737 & 0.098103 & 0.504527 \end{bmatrix} \end{array}.$$

The NSE gain state has highest likelihood with 50%, second highest is fall state with 44.31% and least likelihood is normal state with 5.7%.

The Wipro's share has the highest likelihood with a 50.45% possibility of being in decrement state, second highest likelihood with a 39.73% possibility of being in increment state and least likelihood with a 9.8% possibility of being in remain same state. Hence, it is suggested to purchase the Wipro's shares on the 20th day to make good profits.

6.1.3. Probability distribution for increment state

Considering the above values, the probability distributions of the increment state sequences of one and two days length, observed with Wipro's stock prices are provided below.

Table 1. Probability distributions for increment state in one day and two days length

P (increment)	0	1	2
One day length	0.557954	0.442046	-
Two days length	0.311256	0.493295	0.19545

From Table 1, it is observed that happening of Wipro's increment state is having less likelihood in one day sequence. Further, in a run of two days sequence, the chance of increment in one day is more likely. Hence, it reveals that by observing two days business length, there is more chance for one day in two days run of observation. The required statistical measures for the probability mass function of increment state are presented in Table 2 to observe the nature of probability mass function.

6.1.4. Statistical measures/characteristics for increment state

Considering the above probability distribution values, the statistical measures of increment state sequence of one and two days length, observed with Wipro's stock prices are provided below:

Table 2. Statistical measure for increment state of one and two days length

Statistical measure	One day length	Two days length
Mean (μ_1^1)	0.442046	0.884194
Standard deviation (σ)	0.49663	0.702349
Third central moment (μ_3)	0.028588	0.057126
Skewness (β_1)	0.05447	0.027187
Kurtosis (β_2)	1.05447	2.027187
Coefficient of variation	112.348	79.4338

Considering the above values, the probability distributions of the remain same state sequence of one and two days length, observed with Wipro's stock prices are provided below.

Table 3. Probability distributions for remain same state in one day and two days length

P (remain same)	0	1	2
One day length	0.898749	0.101251	-
Two days length	0.807746	0.182002	0.010252

From Table 3, it is observed that happening of Wipro's remain same state is having less likelihood in one day sequence. Further, in a run of two days sequence, the chance of remain same state in non-happening of the state is more likely. Hence, it reveals that by observing two days business length, there is more chance for non-happening, i.e., Wipro's stock prices are not stable (dynamic). The required statistical measures for the probability mass function of remain same state is presented in Table 4 to observe the nature of probability mass function.

6.1.6. Statistical measures/characteristics for remain same state

Considering the above probability distribution values, the statistical measures of remain same state sequence of one and two days length, observed with Wipro's stock prices are provided below.

Table 4. Statistical measure for remain same state of one and two days length

Statistical measure	One day length	Two days length
Mean (μ_1^1)	0.101251	0.202507
Standard deviation (σ)	0.301661	0.426617
Third central moment (μ_3)	0.072572	0.145146
Skewness (β_1)	6.989108	3.49444
Kurtosis (β_2)	7.989108	5.49444
Coefficient of variation	297.9337	210.6682

6.1.7. Probability distribution for decrement state

Considering the above values, the probability distributions of the decrement state sequence of one and two days length, observed with Wipro's stock prices are provided below:

Table 5. Probability distributions for decrement state in one day and two days length

P (decrement)	0	1	2
One day length	0.543297	0.456703	-
Two days length	0.29523	0.496241	0.208529

From Table 5, it is observed that happening of Wipro's decrement state is less likelihood in one day sequence. Further, in a run of two days sequence, the chance of decrement in one day is more likely. Hence, it reveals that by observing two days business length, there is more chance for one day in two days run of observation. The required statistical measures for the probability mass function of decrement state are presented in Table 6 to observe the nature of probability mass function.

6.1.8. Statistical measures/characteristics for decrement state

Considering the above probability distribution values, the statistical measures of decrement state sequence of one and two days length, observed with Wipro's stock prices, are provided below:

Table 6. Statistical measure for decrement state of one and two days length

Statistical measure	One day length	Two days length
Mean (μ_1^1)	0.456703	0.913299
Standard deviation (σ)	0.498122	0.704444
Third central moment (μ_3)	0.021486	0.043025
Skewness (β_1)	0.030221	0.015148
Kurtosis (β_2)	1.030221	2.015148
Coefficient of variation	109.0691	77.13181

6.2. Findings and recommendations

From Tables 2, 4 and 6, it is observed that the one day sequences, decrement state occurs most frequently (0.456703), suggesting that it is favorable for short-term traders to buy shares for potential returns. In two day sequences, remain same state is less likely (0.202507) compared to increment and decrement states, indicating Wipro's stock is dynamic, and investors should make informed decisions when investing.

In one day sequences, increment and decrement states show higher volatility (0.49663 and 0.498122) compared to remain same state (0.301661). One day sequences are more consistent in remain same state, aiding traders in decision-making. For two days sequences, increment and decrement states exhibit higher volatility (0.702349 and 0.704444) compared to remain same state (0.426617). Two days sequences are more consistent in remain same state, which helps investors make optimal decisions and manage their stock portfolios.

Regarding skewness, the third central moments of the distributions of increment state, remain same state and the decrement state are non-negative. It reveals that all the states of one day trading possess positively skewed distribution. Similarly, the third central moments of all states in the two days length are non-negative, and hence it reveals that all the states of two days trading possess positively skewed distribution.

In one day sequences, increment and decrement states have platykurtic distributions (kurtosis < 3), while remain same state has a leptokurtic distribution (kurtosis > 3). In two days sequences, remain same state likely follows a leptokurtic distribution (kurtosis > 3) as it is more peaked than the other two states.

In one day sequences, remain same state shows high variability (C.V = 297.9337), while decrement state has lower variability. This suggests potential for traders to consider buying Wipro's stock for better returns. In two days sequences, similar trends are observed with remain same state having high variability (C.V = 210.6682) and decrement state showing lower variability. This also indicates an opportunity for traders to consider purchasing Wipro's stock for potential returns.

6.3. Expected returns

The expected long and short-run returns computed utilizing the formula in Kilic [12]. The estimated long-run return is 0.000211317. The expected short-run returns results are presented in Tables 7-9. The following table presents the short-run expected returns for each state from $t = 1$ to $t = 18$ when the returns move into steady state.

Table 7. Expected short-run returns

	Day	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$
State	Increment	0.0078364	0.003577	0.00167	0.00084	0.000482423	0.0003281
	Remain same	-0.003905	-0.00204	-0.000809	-0.00023	1.91921E-05	0.0001285
	Decrement	-0.005043	-0.00199	-0.000731	-0.00019	3.67639E-05	0.0001361

Table 8. Expected short-run returns

	Day	$t = 7$	$t = 8$	$t = 9$	$t = 10$	$t = 11$	$t = 12$
State	Increment	0.000262	0.000233	0.000221	0.000215	0.000213	0.000212
	Remain same	0.000176	0.000196	0.000205	0.000208	0.00021	0.000211
	Decrement	0.000179	0.000197	0.000205	0.000209	0.00021	0.000211

Table 9. Expected short-run returns

	Day	$t = 13$	$t = 14$	$t = 15$	$t = 16$	$t = 17$	$t = 18$
State	Increment	0.000214	0.000215	0.000213	0.000214	0.000212	0.000211
	Remain same	0.000212	0.000214	0.000211	0.000216	0.000218	0.000214
	Decrement	0.000211	0.000211	0.000212	0.000211	0.000211	0.000215

The risk premium of the CAPM obtained from the relation $(r_m - r_f)$, where r_m is the average return of the market for the year estimated from the daily closing price of the Wipro's stock exchange, and r_f is the average risk-free rate of return of the year, the yield on the government treasury bill which is relatively risk-free. The value of β is $0.938211102 \simeq 1$ and expected return was found to be 0.006786667. It results that the Wipro's share expected risk is proportional to the market risk. Thus the investor may expect and get the optimal returns by investing on Wipro's stock.

6.4. Predicted stock values

The share values of Wipro Company are predicted by using equation 5.4 placed in Subsection 5.3. The predicted share values of a company will be helpful to the short-term traders. The predicted share value may answer the questions like (i) what is stock value in the increment state on the 249th day? (ii) what is the stock value in the remain same state on 249th day? And also (iii) what is the stock value in decrement state on 249th day? Tables 10-12 represent the predicted values of the Wipro's share prices.

Table 10. Predicted stock values

	Day	249th day	250th day	251st day	252nd day	253rd day	254th day
State	Increment	592.3055	594.42433	595.4169	595.9173	596.2048	596.4004
	Remain same	585.4052	584.21129	583.7388	583.6026	583.6138	583.6888
	Decrement	584.736	583.57131	583.1445	583.0313	583.0528	583.1321

Table 11. Predicted stock values

	Day	255th day	256th day	257th day	258th day	259th day	260th day
State	Increment	596.5565	596.6955	596.8271	596.9556	597.0828	597.2094
	Remain same	583.7913	583.9057	584.0252	584.147	584.2697	584.3928
	Decrement	583.2365	583.3516	583.4714	583.5931	583.7158	583.8389

Table 12. Predicted stock values

	Day	261st day	262nd day	263rd day	264th day	265th day	266th day
State	Increment	597.3357	597.462	597.5883	597.7146	597.8409	597.9673
	Remain same	584.5163	584.6397	584.7633	584.8868	585.0104	585.134
	Decrement	583.9622	584.0855	584.2089	584.3324	584.4559	584.5794

The above results may helpful to the investors like short-term business traders and portfolio managers. If the traders share is in risk, they may take some remedial measure to optimize their stock without loss.

6.5. Chi-square goodness of fit

The developed probability distribution model on goodness of fit is verified through chi-square test. Here, H_0: there is no significance difference between observed and expected closing prices, H_1: there is significance difference between observed and expected closing prices. The χ^2 critical value is considered at 5% level of significance. The chi-square test statistic value is 9.625781033 and chi-square table value at 17 degrees of freedom is 27.587. Since $\chi^2_{\text{test statistic}} < \chi^2_{\text{critical value}}$, it is observed that the null hypothesis cannot be rejected and the fitted model is good enough.

7. Conclusion

The current research work is on developing HMM for daily closing price of the NSE and Wipro Company. The transition states of NSE are considered as hidden state influencing the observed states of Wipro Company. The parameters of HMM like IPV, TPM, and OPM/EPM are obtained by assuming the discrete Markov chains within hidden states and between hidden to observed states, respectively. This study has formulated the different probability distributions for increment, remain same, and decrement states. Model behaviour of all states is explored through the explicit mathematical relations of different statistical characteristics and Pearson's coefficients, presented in Sections 3 and 4. Sensitivity analysis is carried out through the real-time data sets of NSE and Wipro Company for understanding the characteristic of the developed model, which is presented in Subsections 6.1 and 6.2. These results will be useful for short-term business investors for finding the indicators of when to sell and when to purchase the share of Wipro Company by observing the chance of its emission states. The results mentioned in Subsection 6.3 show that the overall average of returns of the stock (0.000211317) will be realized in the long-run (after 20th day). The predicted share values of Wipro Company are presented in Tables 10-12 in Subsection 6.4, these results may give a good indication to the traders for optimizing their share by choosing the options of selling/purchasing.

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