Two Hidden Layer's Markov Model for Studying Type 2 Diabetes

Tirupati Rao padi
pondicherry university
Surnam Narendra (surnamnarendra@gmail.com)
Pondicherry University https://orcid.org/0009-0000-5687-5912

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Two Hidden Layer's Markov Model for Studying Type 2 Diabetes

Padi Tirupati Rao[1] and Surnam Narendra[2*†]

[1,2]Department of Statistics, Pondicherry University, Kalapet, Pondicherry, 605014, Pondicherry, India.

*Corresponding author(s). E-mail(s): surnamnarendra@gmail.com;
Contributing authors: drtrpadi@gmail.com;
†These authors contributed equally to this work.

Abstract

Diabetes is a metabolic disorder that can be regulated through proper physical activity and diet management. This paper is about developing a stochastic model by using Layered Hidden Markov Model (LHMM) for diabetes. In LHMM, we considered two hidden layers and one emission state layer. Here, we find out the probability of emission state sequence by the forward algorithm. Further, we derived probability mass function for one-state, two-state and three-state sequences for non-diabetes and diabetes emission states. We also derived the explicit relations for mean, variance and other Pearson's coefficients of Skewness and Kurtosis of the formulated probability distribution. The model behaviour is studied in detail with real-time data on diabetes from secondary data sources. This study considered three variables, namely (i) physical activity, (ii) obesity category based on BMI, and (iii) having diabetes. It is assumed that diabetes is influenced by obesity, and obesity is influenced by physical activity. After a thorough analysis of data, it is observed that (i) from one day sequence, the state of non-diabetes is more likely, (ii) from the two-day sequence, the chances of the probability for two consecutive occurrences of the emission state of non-diabetes is 0.50; and (iii) from the three-day sequence, the chance of three consecutive occurrences of the emission state non-diabetes is around 0.38. From the transition and emission probability matrices, we found that those who are doing physical activity daily have less chance of obesity, and those who are not doing physical activity have a high chance of obesity, and later, they will be getting the risk of diabetes

Keywords: Diabetes, Layer Hidden Markov Model (LHMM), Probability Distribution, Obesity, Physical Activity.

1 Introduction

Diabetes is a metabolic disorder which can be regulated through the proper management of physical activity, diet and some extent of using drugs. It is a threefold management strategy for diabetes care. Glucose metabolism is used to maintain the normal blood glucose level within the 70-110 mg/dl range in a healthy person. Glucose metabolism is influenced by factors like insulin secretion by the pancreas through beta and gamma cells. The rate of glucose metabolism among type-2 diabetes patients will be sullen due to the lack of sufficient insulin secretion in the blood. Healthy levels of weight and BMI are also playing a vital role in the maintenance of normal glucose levels as physical activity and BMI are regulating factors. Increasing the insulin secretion levels through the pancreas is influenced by the BMI, and physical activity can be achieved by properly understanding the pancreas's functioning and metabolism rate.

According to International Diabetes Foundation (IDF) 2021 report, 10.5% of the adult population has diabetes, and they have predicted that by 2045, every 1 in 8 adults may become diabetic. World Health Organization declared that about 1.5 million deaths were caused by diabetes, kidney diseases caused by diabetes and CVD in 2019. India ranks second in global for diabetes epidemics after nearby country China. There is much-reported research work on diabetes. This study focused on developing the layer-hidden Markov model (LHMM) for diabetes to understand the relationship between physical activity, obesity and diabetes. It has considered two hidden layered variables namely (i) physical activity and (ii) obesity, and there is a single layer of emission states on diabetes. The construction of LHMM is to get the explicit mathematical relations for probabilities of the emission states with different lengths of sequences. The activity is extended to derive the probability distributions of different states and lengths of the sequences. The statistical characteristics are obtained in terms of mathematical relations for the formulated probability distributions. In order to verify the validity of the model and understand the applied utilities of the developed model, secondary data on diabetes is used from Neha Prerna Tigga and Shruti Garg in 2019.

Markov model is a stochastic model where the current state depends only on the previous immediate state introduced by Andrei A. Markov (1906). Introduced the Hidden Markov Model [1]. Provided a comprehensive overview of HMMs and their application in speech recognition [2]. Used LHMM on a robust tokenizer that can correct spelling mistakes and recover from segmentation errors [3]. Provided the layered probabilistic representations using hidden Markov models for performing, sensing, learning, and inference tasks at multiple levels of temporal granularity [4]. Used layered hidden Markov models to diagnose the states of a user's activity based on real-time streams of evidence from video, audio, and computer interactions [5]. Mentioned that LHMM is suitable for modelling te-le-operation trajectory-tracking tasks [6]. Demonstrated the hierarchical architecture decouples the motion analysis into different temporal granularity levels by LHMM [7]. Proposed the first layer of HMMs detects short primitive motions with direct targets [8]. The cross-layer inferences were used to design CLHMM to strengthen the reasoning efficiency and reduce the computational complexity [9]. Identified that the double-layer HMM can recognize the driving intention and predict the driving behaviour accurately and efficiently [10]. Applied trained HMM for Diabetic Retinopathy screen images with genetics [11]. Proposed a model that achieves significantly better recognition performance than the other models [12]. Applied the layered hidden Markov model and successfully used it with physiological data from co-contracting arm muscles to achieve accurate intent prediction during task operations [13]. Applied HMM for estimating the prevalence of type-2 diabetes through $HbA1c \geq 6.5$ [14]. Applied three multilevel HMM for type 1 diabetes and estimated the probabilities between generations from parents and children [15].

After observing the reported literature on HMM for diabetes, it is evident that maximum research studies are on single one-layer HMM such as using HbA1c levels, BMI, sugar levels, etc. for diabetes. However, diabetes is affected by more than one causing factor. The study of diabetes impact shall be influenced by more than one hidden factor hence, in order to get the rational relations of emission states, that shall be considered with the impact of more than one causing hidden factor. Therefore applying the general single-layered HMM may not be appropriate. This limitation can be avoided by considering the multi-layered HMM. The core activity of this study is the formation and characterization of Probability distributions of different emission states that are influenced by two-layer hidden states. Deriving the explicit mathematical relations of statistical characteristics is an extended activity. Understanding the model behaviour with real-time data and the performance of sensitivity analysis on the fluctuations of diabetic status is also the core motivation behind this study.

2 Two layer hidden Markov model

The hidden Markov model (HMM) is a stochastic model used to find the chance of hidden states that influence the emission state's sequence. For example, HMM will be useful in finding the probability of observing the patient in the state of diabetes when he/she is in the different states of HbA1c. Diagnoists can classify whether patients are in a healthy, pre-diabetic or diabetic state based on the emission state's probabilities. The specified categorical states based on HbA1c (i) $4 \leq HbA1c \leq 5.6$; (ii) $5.7 \leq HbA1c < 6.5$; and (iii) HbA1c = 6.5+ are considered as the hidden states. Classified states on the condition of the patients like (i) Healthy; (ii) Pre-Diabetic and (iii) Diabetic are the emission states.

HMM are widely used in computer science, engineering, Computational Biology, Machine learning, AI, etc. Usually, HMMs are of one-layer hidden states and one-layer emission states. For modelling different stochastic processes in the natural language, the word Hierarchical Hidden Markov Model (HHMM) was used by [16]. Further, when the data have mixed up with different combinations of variables, it is more appropriate to study the observed states with the Layered Hidden Markov Model (LHMM).

The Layer Hidden Markov Model (LHMM) is an HMM based on data that can interchange our model, but it should satisfy the assumptions of the Markov chain. So, in this paper, we made an attempt on LHMM for diabetes because diabetes is influenced by many factors, and each factor may be considered as one layer. So based on this motive we have initiated the study with two LHMM where both the layers have two hidden states each and two emission states. This model has considered the emission states as (i) non-diabetic and (ii) diabetic; hidden layer 2 is with states of (i) having obesity and (ii) not having obesity; influenced by the hidden layer 1 with the states (i) presence of physical activity and (ii) absence of physical activity. The figure [1] presents the functional schematic diagram.

2.1 Assumption and notations of the model

Let $\lambda = (\pi, A, B, C, D)$ be the proposed model to develop.
Let π be the Initial probability vector of the hidden states (layer-1).
Let $A =$ be the Transition probability matrix among the states of the first hidden layer.
Let B be the Connection probability matrix among the states from the first to the second hidden layer.
Let C be the Transition probability matrix among the states of the second hidden layer.
Let D be the Emission probability matrix among the observed states (from the second hidden layer's states to the observed states).

2.2 Mathematical description

The mathematical description for the proposed two-layer hidden Markov model is denoted as $\lambda = (\pi, A, B, C, D)$ as follows.

2.2.1 Initial probability vector

Initial probabilities are the starting probabilities of the model. In this proposed model, we have two hidden layers for initial probabilities. However, the probabilities of the first hidden layer are only considered for the study purpose. Letπ_i be the initial probability of the ith state in the first hidden layer. As we have considered two hidden states in the first layer, i=1,2. $\pi_i \geq 0$, and $\sum_{i=1}^{2} \pi_i = 1$.Hence, i represents the number of hidden states in the first layer, $0 \leq \pi_i \leq 1$.

2.2.2 Intra-layer transition probability matrices (A & C)

A and C are the TPMs with the transition probabilities within the states of hidden layers 1 and 2 respectively.

Within layer-1: The TPM deal with the intra-transitions among the hidden states of the first layer. Let X_n be the value (state) of n^{th} trial in the Hidden layer-1. Further, $A = [a_{ij}]$;$aij = Pr\{X_{n+1} = j|X_n = i\}$; $\sum_{j=1}^{2} a_{ij} = 1 \, \forall \, i = 1, 2$ .where i and j are the transitions within a hidden state of layer-1 and$0 \leq a_{ij} \leq 1$.

Within layer-2: The TPM deal with the intra-transitions among the hidden states of the second layer. Let Y_n be the value (state) of n^{th} trial in the Hidden layer-2. Further, $C = [c_{kl}]$; $c_{kl} = Pr\{Y_{n+1} = l|Y_n = k\}$; $\sum_{l=1}^{2} c_{kl} = 1 \,\forall\, k = 1, 2.$where k and l are the transitions within hidden states of layer-2 and $0 \leq c_{kl} \leq 1$.

2.2.3 Inter-layer transition (Connecting) probability matrix (B):

These matrices deal with the inter-layer connecting probabilities. B is the connecting transition probability matrix between layer-1's hidden states & layer-2's hidden states. D is the Emission transition matrix between the hidden states of layer-2 and the emission states of the observed variable. $B = [b_{jk}]$; $bjk = Pr\{Yn + 1 = k|Xn = j\}$; $\sum_{k=1}^{2} b_{jk} = 1 \,\forall\, j = 1, 2.$where j and k are the transitions between j^{th} hidden state of layer-1 to k^{th} hidden state of layer-$2.0 \leq b_{jk} \leq 1$.

2.2.4 Emission probabilities

Emission probabilities are the observed state probabilities from the latest hidden layer. $D = [d_{lm}]$ $d_{lm} = Pr\{Z_{n+1} = m|Y_n = l\}$; $\sum_{m=1}^{2} d_{lm} = 1 \,\forall\, l = 1, 2.$where l and m are the transitions between l^{th} hidden state of layer-2 to m^{th} emission state of observed variable, and$0 \leq d_{lm} \leq 1$.

2.3 Schematic diagram for two-layer HMM

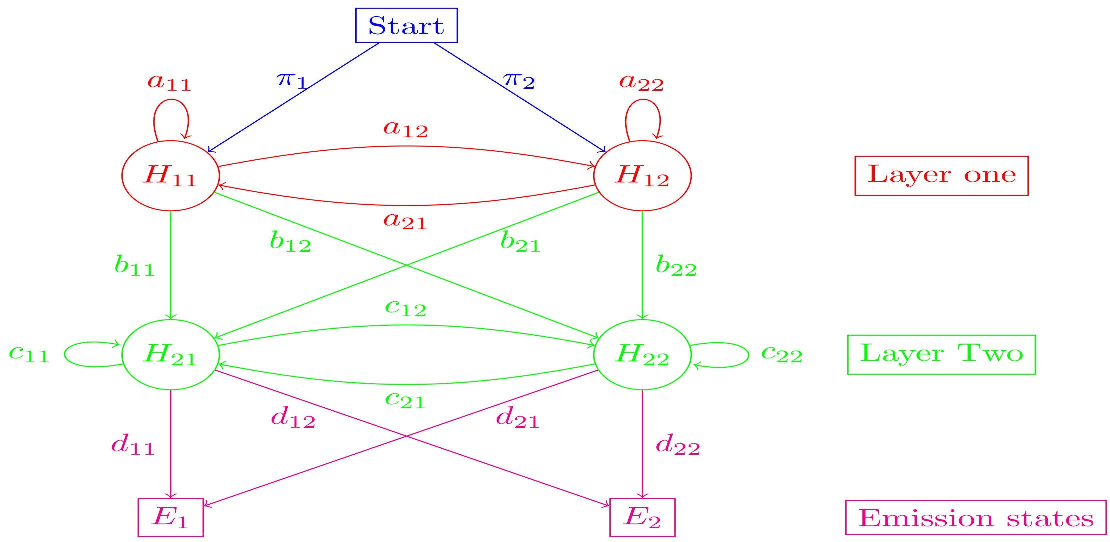

Fig. 1 Layered hidden Markov model with two hidden states and one emission layer

Hidden states and Layers are denoted by

$$H_{ij} \,\forall\, i = 1, 2 \,\&\, j = 1, 2$$

where i is the layer number and j is the state number.

3 Emission state sequence and its probability distribution

This section deals with a forward algorithm to formulate probability distributions for the emission state sequences with different lengths (say one, two and three). Using HMM, we can find the probability of emission states sequence by forward algorithm, best-hidden states sequence path and parameter estimation by Baum-Welch algorithm. Our model has two hidden layers with two states each and one emission layer with two states. Once the probabilities of different lengthened sequences are obtained, it will facilitate finding the probability distributions of that specific sequence. To submit on more clear lines, suppose there are two emission states, the sequence length is three, then the total possible events are eight and finding the probability distribution needs to find the probabilities of each of these eight exclusive events. This study focused on formulating the probability mass function of the two emission states namely 'non-diabetes state' and 'diabetes state' with the length of one-state, two-state and three-state sequences. There are altogether six probability mass functions formulated and verified for validity. Further, the mathematical expressions for statistical characteristics like mean, variance, Skewness and Kurtosis are derived. The mathematical model while formulating considers the notations of E_1 means non-diabetes, E_2 means diabetes, H_{21} means not having obesity and H_{22} means having obesity and H_{11} means not doing physical activity and H_{12} means doing physical activity.

3.1 Forward algorithm for two hidden layers and one emission layer

Algorithm 1 Forward algorithm for two hidden layers and one emission layer

Require:

λ: Sequence of observed emissions (length T)
$N1$: Number of states in the first hidden layer
$N2$: Number of states in the second hidden layer
π: Initial state probability vector for the first hidden layer
A: Transition probability matrix for the first hidden layer
B: Connection probability matrix from the first to the second hidden layer
C: Transition probability matrix for the second hidden layer
D: Emission probability matrix for the second hidden layer

Ensure: $P(\lambda \mid \text{model})$: Probability of observing the sequence λ given the two-layer HMM model

Initialize α matrix of size $T \times N1 \times N2$
$\alpha[1][i][j] \leftarrow \pi 1[i] \times B[i][j] \times D[j][O[1]]$ for $i = 1$ to $N1$ and $j = 1$ to $N2$
for $t \leftarrow 2$ to T **do**
    for $i \leftarrow 1$ to $N1$ **do**
        for $j \leftarrow 1$ to $N2$ **do**
            $\alpha[t][i][j] \leftarrow \sum_{\text{prev}_i=1}^{N1} \sum_{\text{prev}_j=1}^{N2} \alpha[t-1][\text{prev}_i][\text{prev}_j] \times A[\text{prev}_i][i] \times C[\text{prev}_j][j] \times D[j][O[t]]$
        end for
    end for
end for
$P(\lambda \mid \text{model}) \leftarrow \sum_{i=1}^{N1} \sum_{j=1}^{N2} \alpha[T][i][j]$

3.2 Probability of one emission state sequence and its probability distribution

The probability of one emission state sequence is given below using the forward algorithm with a graphical representation.

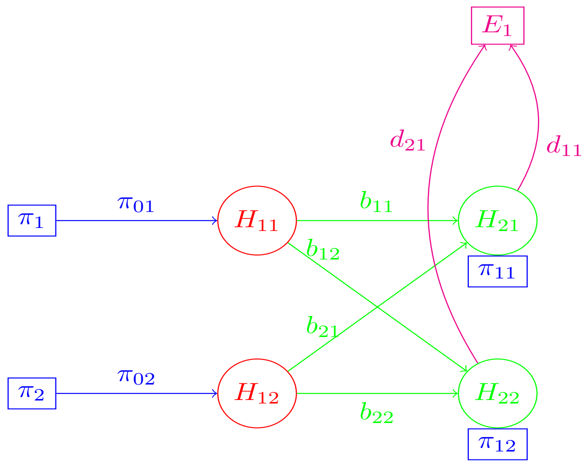

Fig. 2 Foreword algorithm for LHMM with one emission state

The probability for one emission state sequence from the figure [2] is given by

$$P(E_i) = \sum_{j_1=1}^{2} d_{j_1 i} \sum_{j_2=1}^{2} \pi_{0j_2} b_{j_2 j_1} \; ; \forall \, i = 1, 2 \tag{1}$$

3.2.1 Probability distribution & statistical characteristics of emission state (Non-Diabetes) with sequence length one

Let's assume that we have the following one-state sequence, i.e. E_1 and emission states are E_1, E_2. So the total number of possible cases is 2. Let $X_1(\omega)$ =no.of E_1's in emission state sequence, then by using the probability of sequence from the equation [1], pmf is given by.

$$P(X_1(\omega) = x_1) = \begin{cases} \sum_{j_1=1}^{2} d_{j_1 2}\alpha & x_1 = 0 \\ \sum_{j_1=1}^{2} d_{j_1 1}\alpha & x_1 = 1 \end{cases} \tag{2}$$

where

$$\alpha = \sum_{j_2=1}^{2} \pi_{j_2} b_{j_2 j_1}$$

Statistical Characteristics

$$\mu_r^1 = \sum_{j_1=1}^{2} d_{j_1 1}\alpha \, \forall \, r = 1, 2, 3, 4 \tag{3}$$

$$\mu_2 = \sum_{j_1=1}^{2} d_{j_1 1}\alpha(1 - \sum_{j_1=1}^{2} d_{j_1 1}\alpha) \tag{4}$$

$$\mu_3 = \sum_{j_1=1}^{2} d_{j_1 1}\alpha\{(1 - 2\sum_{j_1=1}^{2} d_{j_1 1}\alpha)(1 - \sum_{j_1=1}^{2} d_{j_1 1}\alpha)\} \tag{5}$$

$$\mu_4 = \sum_{j_1=1}^{2} d_{j_1 1}\alpha(\sum_{j_1=1}^{2} d_{j_1 1}\alpha - 1)\{3\sum_{j_1=1}^{2} d_{j_1 1}\alpha - 3(\sum_{j_1=1}^{2} d_{j_1 1}\alpha)2 - 1\} \tag{6}$$

$$\beta_1 = (1 - 2\sum_{j_1=1}^{2} d_{j_1 1}\alpha)^2(\sum_{j_1=1}^{2} d_{j_1 1}\alpha(1 - \sum_{j_1=1}^{2} d_{j_1 1}\alpha))^{-1} \tag{7}$$

$$\beta_2 = (3(\sum_{j_1=1}^{2} d_{j_1 1}\alpha)^2 - 3\sum_{j_1=1}^{2} d_{j_1 1}\alpha + 1)\{\sum_{j_1=1}^{2} d_{j_1 1}\alpha(1 - \sum_{j_1=1}^{2} d_{j_1 1}\alpha)\}^{-1} \tag{8}$$

3.2.2 Probability distribution & statistical characteristics of emission state (Diabetes) with sequence length one

Let's assume that we have the following one-state sequence, i.e E_2 and emission states are E_1, E_2. So the total number of possible cases is 2. Let $X_2(\omega)$ =no.of E_2's in emission state sequence, then by using the probability of sequence from the equation [1], pmf is given by.

$$P(X_2(\omega) = x_2) = \begin{cases} \sum_{j_1=1}^{2} d_{j_1 1}\alpha & x_1 = 0 \\ \sum_{j_1=1}^{2} d_{j_1 2}\alpha & x_1 = 1 \end{cases} \tag{9}$$

where

$$\alpha = \sum_{j_2=1}^{2} \pi_{0 j_2} b_{j_2 j_1}$$

Statistical Characteristics

$$\mu_r^1 = \sum_{j_1=1}^{2} d_{j_1 2}\alpha \; \forall \, r = 1, 2, 3, 4 \tag{10}$$

$$\mu_2 = \sum_{j_1=1}^{2} d_{j_1 2}\alpha(1 - \sum_{j_1=1}^{2} d_{j_1 2}\alpha) \tag{11}$$

$$\mu_3 = \sum_{j_1=1}^{2} d_{j_1 2}\alpha\{(1 - 2\sum_{j_1=1}^{2} d_{j_1 2}\alpha)(1 - \sum_{j_1=1}^{2} d_{j_1 2}\alpha)\} \tag{12}$$

$$\mu_4 = \sum_{j_1=1}^{2} d_{j_1 2}\alpha(\sum_{j_1=1}^{2} d_{j_1 2}\alpha - 1)\{3\sum_{j_1=1}^{2} d_{j_1 2}\alpha - 3(\sum_{j_1=1}^{2} d_{j_1 2}\alpha)2 - 1\} \tag{13}$$

$$\beta_1 = (1 - 2\sum_{j_1=1}^{2} d_{j_1 2}\alpha)^2(\sum_{j_1=1}^{2} d_{j_1 2}\alpha(1 - \sum_{j_1=1}^{2} d_{j_1 2}\alpha))^{-1} \tag{14}$$

$$\beta_2 = (3(\sum_{j_1=1}^{2} d_{j_1 2}\alpha)^2 - 3\sum_{j_1=1}^{2} d_{j_1 2}\alpha + 1)\{\sum_{j_1=1}^{2} d_{j_1 2}\alpha(1 - \sum_{j_1=1}^{2} d_{j_1 2}\alpha)\}^{-1} \tag{15}$$

3.3 Probability of two emission state sequence and its probability distribution

The probability of two emission state sequences is given below using the forward algorithm with a graphical representation.

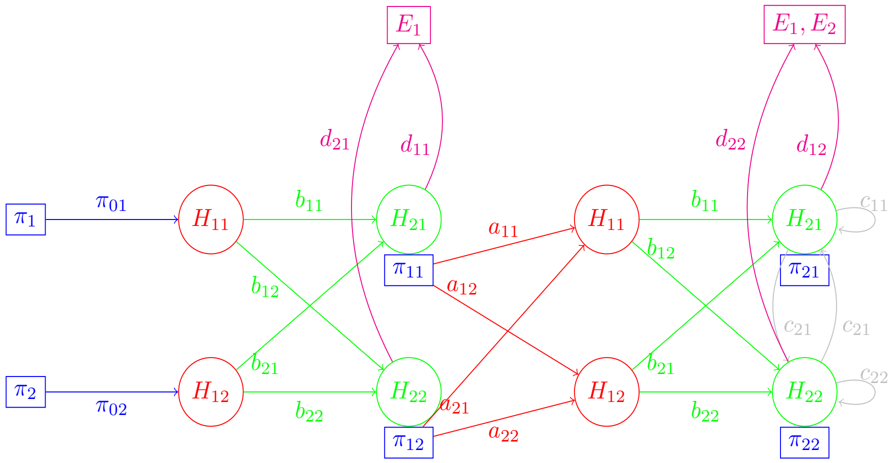

Fig. 3 Foreword algorithm for LHMM with two emission states

The mathematical expression for obtaining the probability of two emission state sequences from the figure [3] is given by

$$P(E_{i_1}E_{i_2}) = \sum_{j_1=1}^{2}\sum_{j_2=1}^{2} d_{j_1 i_1} d_{j_2 i_2} c_{j_1 j_2} \sum_{j_3=1}^{2}\sum_{j_4=1}^{2} \pi_{1 j_3} a_{j_3 j_4} b_{j_4 j_1} \; ; \forall\, i_1, i_2 = 1, 2 \tag{16}$$

3.3.1 Probability distribution & statistical characteristics of emission state (Non-Diabetes) with sequence length two

Let's assume that we have the following one-state sequence, i.e. E_1, E_2 and emission states are E_1, E_2. So the total number of possible cases is 2. Let $X_3(\omega)$ =no.of E_1's in emission state sequence, then by using the probability of sequence from the [16], pmf is given by.

$$P(X_3(\omega) = x_3) \begin{cases} \sum_{j_1=1}^{2}\sum_{j_2=1}^{2} d_{j_1 2} d_{j_2 2}\alpha & x_2 = 0 \\ \sum_{j_1=1}^{2}\sum_{j_2=1}^{2} (d_{j_1 1} d_{j_2 2} + d_{j_1 2} d_{j_2 1})\alpha & x_2 = 1 \\ \sum_{j_1=1}^{2}\sum_{j_2=1}^{2} d_{j_1 1} d_{j_2 1}\alpha & x_2 = 2 \end{cases} \tag{17}$$

where,

$$\alpha = \sum_{j_3=1}^{2} \sum_{j_4=1}^{2} \pi_{1j_3} a_{j_3j_4} b_{j_4j_1} c_{j_1j_2}$$

Statistical Characteristics

$$\mu_r^1 = \theta_1 + 2^r\theta_2 \tag{18}$$

$$where\, \theta_1 = \sum_{j_1=1}^{2} \sum_{j_2=1}^{2} (d_{j_11}d_{j_22} + d_{j_12}d_{j_21})\alpha \,\&\, \theta_2 = \sum_{j_1=1}^{2} \sum_{j_2=1}^{2} d_{j_11}d_{j_21}\alpha \tag{19}$$

$$\mu_2 = \theta_1 - \theta_1^2 + 4\theta_2 - 4\theta_1\theta_2 - 4\theta_2^2 \tag{20}$$

$$\mu_3 = (-1 + \theta_1 + 2\theta_2)(2\theta_1^2 + 8(-1 + \theta_2)\theta_2 + \theta_1(-1 + 8\theta_2)) \tag{21}$$

$$\mu_4 = \theta_1 - 4\theta_1^2 + 6\theta_1^3 - 3\theta_1^4 + 16\theta_2 - 440\theta_1\theta_2 + 36\theta_1^2\theta_2 - 24\theta_1^3\theta_2 - 64\theta_2^2 + 72\theta_1\theta_2^2 + 48\theta_2^3 - 96\theta_1\theta_2^3 - 48\theta_2^4 \tag{22}$$

$$\beta_1 = \frac{(-1 + \theta_1 + 2\theta_2)^2(2\theta_1^2 + 8(-1 + \theta_2)\theta_2 + \theta_1(-1 + 8\theta_2))^2}{(\theta_1^2 + 4(\theta_2 - 1)\theta_2 + \theta_1(4\theta_2 - 1))^3} \tag{23}$$

$$\beta_2 = \sqrt{\frac{\theta_1 - 4\theta_1^2 + 6\theta_1^3 - 3\theta_1^4 + 16\theta_2 - 440\theta_1\theta_2 + 36\theta_1^2\theta_2 - 24\theta_1^3\theta_2 - 64\theta_2^2 + 72\theta_1\theta_2^2 + 48\theta_2^3 - 96\theta_1\theta_2^3 - 48\theta_2^4}{(\theta_1 - \theta_1^2 + 4\theta_2 - 4\theta_1\theta_2 - 4\theta_2^2)2}} \tag{24}$$

3.3.2 Probability distribution & statistical characteristics of emission state (Diabetes) with sequence length two

Let's assume that we have the following one-state sequence, i.e. E_1, E_2 and emission states are E_1, E_2. So the total number of possible cases is 2. Let $X_4(\omega)$ =no.of E_2's in emission state sequence, then by using the probability of sequence from the equation [16], pmf is given by.

$$P(X_4(\omega) = x_4) \begin{cases} \sum_{j_1=1}^{2} \sum_{j_2=1}^{2} d_{j_11}d_{j_21}\alpha & x_2 = 0 \\ \sum_{j_1=1}^{2} \sum_{j_2=1}^{2} (d_{j_11}d_{j_22} + d_{j_12}d_{j_21})\alpha & x_2 = 1 \\ \sum_{j_1=1}^{2} \sum_{j_2=1}^{2} d_{j_12}d_{j_22}\alpha & x_2 = 2 \end{cases} \tag{25}$$

$$\alpha = \sum_{j_3=1}^{2} \sum_{j_4=1}^{2} \pi_{1j_3} a_{j_3j_4} b_{j_4j_1} c_{j_1j_2}$$

Statistical Characteristics

$$\mu_r^1 = \theta_1 + 2^r\theta_2 \tag{26}$$

$$where\, \theta_1 = \sum_{j_1=1}^{2} \sum_{j_2=1}^{2} (d_{j_11}d_{j_22} + d_{j_12}d_{j_21})\alpha \& \theta_2 = \sum_{j_1=1}^{2} \sum_{j_2=1}^{2} d_{j_12}d_{j_22}\alpha \tag{27}$$

$$\mu_2 = \theta_1 - \theta_1^2 + 4\theta_2 - 4\theta_1\theta_2 - 4\theta_2^2 \tag{28}$$

$$\mu_3 = (-1+\theta_1+2\theta_2)(2\theta_1^2+8(-1+\theta_2)\theta_2+\theta_1(-1+8\theta_2)) \tag{29}$$

$$\mu_4 = \theta_1 - 4\theta_1^2 + 6\theta_1^3 - 3\theta_1^4 + 16\theta_2 - 440\theta_1\theta_2 + 36\theta_1^2\theta_2 - 24\theta_1^3\theta_2 - 64\theta_2^2 + 72\theta_1\theta_2^2 + 48\theta_2^3 - 96\theta_1\theta_2^3 - 48\theta_2^4 \tag{30}$$

$$\beta_1 = \frac{(-1+\theta_1+2\theta_2)^2(2\theta_1^2+8(-1+\theta_2)\theta_2+\theta_1(-1+8\theta_2))^2}{(\theta_1^2+4(\theta_2-1)\theta_2+\theta_1(4\theta_2-1))^3} \tag{31}$$

$$\beta_2 = \sqrt{\frac{\theta_1 - 4\theta_1^2 + 6\theta_1^3 - 3\theta_1^4 + 16\theta_2 - 440\theta_1\theta_2 + 36\theta_1^2\theta_2 - 24\theta_1^3\theta_2 - 64\theta_2^2 + 72\theta_1\theta_2^2 + 48\theta_2^3 - 96\theta_1\theta_2^3 - 48\theta_2^4}{(\theta_1 - \theta_1^2 + 4\theta_2 - 4\theta_1\theta_2 - 4\theta_2^2)2}} \tag{32}$$

3.4 Probability of three emission state sequence and its probability distribution

The forward algorithm concept gives the probability of three emission state sequences.

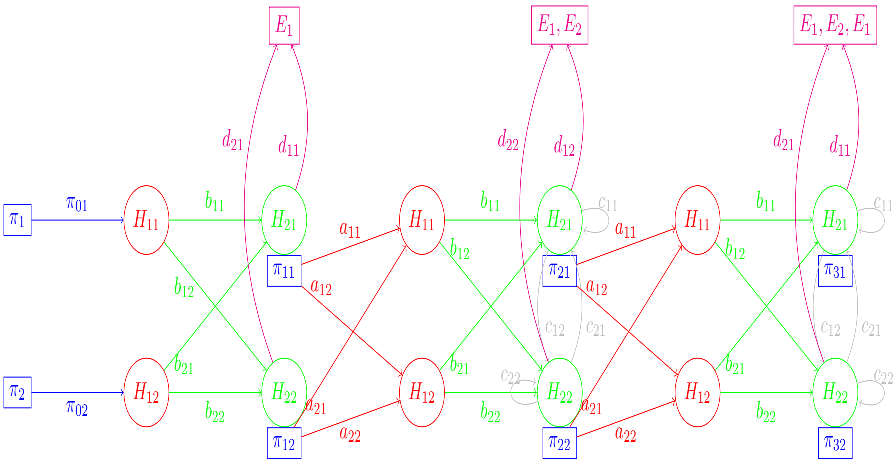

Fig. 4 Forward Algorithm for Three States

The probability sequence for three emission state sequences from the figure [4] is given by.

$$P(E_{i_1}E_{i_2}E_{i_3}) = \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} d_{j_1i_1}d_{j_2i_2}d_{j_3i_3}c_{j_1j_2}c_{j_2j_3} * \sum_{j_4=1}^{2}\sum_{j_5=1}^{2}\sum_{j_6=1}^{2}\pi_{2j_4}a_{j_4j_5}a_{j_5j_6}b_{j_6j_1}; \forall i_1, i_2, i_3 = 1,2,3 \tag{33}$$

3.4.1 Probability distribution & statistical characteristics of emission state (Non-Diabetes) with sequence length three

Let's assume that we have the following one-state sequence, i.e. E_1, E_2, E_1 and emission states are E_1, E_2. So the total number of possible cases is 2. Let $X_4(\omega)$ =no.of E_1's in emission state sequence, then by using the probability of sequence from the equation [33], pmf is given by.

$$P(X_5(\omega) = x_5) = \begin{cases} \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} d_{j_1 2} d_{j_2 2} d_{j_3 2}\alpha & x_3 = 0 \\ \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} (d_{j_1 1} d_{j_2 2} d_{j_3 2} + d_{j_1 2} d_{j_2 1} d_{j_3 2} + d_{j_1 2} d_{j_2 2} d_{j_3 1})\alpha & x_3 = 1 \\ \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} (d_{j_1 1} d_{j_2 1} d_{j_3 2} + d_{j_1 2} d_{j_2 1} d_{j_3 1} + d_{j_1 1} d_{j_2 2} d_{j_3 1})\alpha & x_3 = 2 \\ \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} d_{j_1 1} d_{j_2 1} d_{j_3 1}\alpha & x_3 = 3 \end{cases} \tag{34}$$

where

$$\alpha = \sum_{j_4=1}^{2}\sum_{j_5=1}^{2}\sum_{j_6=1}^{2} \pi_{2 j_4} a_{j_4 j_5} a_{j_5 j_6} b_{j_6 j_1} c_{j_1 j_2} c_{j_2 j_3}$$

Statistical Characteristics

$$\mu_r^1 = \theta_1 + 2^r\theta_2 + 3^r\theta_3 \ , \forall r = 1, 2, 3, 4 \tag{35}$$

$$where, \ \theta_1 = \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} (d_{j_1 1} d_{j_2 2} d_{j_3 2} + d_{j_1 2} d_{j_2 1} d_{j_3 2} + d_{j_1 2} d_{j_2 2} d_{j_3 1})\alpha \tag{36}$$

$$\theta_2 = \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} (d_{j_1 1} d_{j_2 1} d_{j_3 2} + d_{j_1 2} d_{j_2 1} d_{j_3 1} + d_{j_1 1} d_{j_2 2} d_{j_3 1})\alpha \tag{37}$$

$$\theta_3 = \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} d_{j_1 1} d_{j_2 1} d_{j_3 1}\alpha \tag{38}$$

$$\mu_2 = \theta_1 - \theta_1^2 + 4\theta_2 - 4\theta_1\theta_2 - 4\theta_2^2 + 9\theta_3 - 6\theta_1\theta_3 - 12\theta_2\theta_3 - 9\theta_3^2 \tag{39}$$

$$\begin{aligned} \mu_3 = {} & 2\theta_1^3 + 16\theta_2^3 + 24\theta_2^2(-1 + 3\theta_3) + 3\theta_1^2(-1 + 4\theta_2 + 6\theta_3) + 27\theta_3(1 - 3\theta_3 + 2\theta_3^2) \\ & + 2\theta_2(4 - 45\theta_3 + 54\theta_3^2) + \theta_1(1 + 24\theta_2^2 - 36\theta_3 + 54\theta_3^2 + 18\theta_2(-1 + 4\theta_3)) \end{aligned} \tag{40}$$

$$\begin{aligned} \mu_4 = {} & -3\theta_1^4 - 48\theta_2^4 - 96\theta_2^3(-1 + 3\theta_3) - 6\theta_1^3(-1 + 4\theta_2 + 6\theta_3) - 8\theta_2^2(8 - 63\theta_3 + 81\theta_3^2) - 81\theta_3(-1 + 4\theta_3 - 6\theta_3^2 \\ & - 8\theta_2(-2 + 39\theta_3 - 108\theta_3^2 + 81\theta_3^3) - 2\theta_1^2(2 + 36\theta_2^2 - 45\theta_3 + 81\theta_3^2 + 12\theta_2(-2 + 9\theta_3)) \\ & - \theta_1(-1 + 96\theta_2^3 + 120\theta_3 - 378\theta_3^2 + 324\theta_3^3 + 24\theta_2^2(-5 + 18\theta_3) + 8\theta_2(5 - 54\theta_3 + 81\theta_3^2)) \end{aligned} \tag{41}$$

3.4.2 Probability distribution & statistical characteristics of emission state (Diabetes) with sequence length three

Let's assume that we have the following one-state sequence, i.e. E_1, E_2, E_1 and emission states are E_1, E_2. So the total number of possible cases is 2. Let $X_6(\omega)$ =no.of E_2's in emission state sequence, then by using the probability of sequence from the equation

[33], pmf is given by.

$$P(X_6(\omega)=x_6)=\begin{cases}\sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} d_{j_1 1}d_{j_2 1}d_{j_3 1}\alpha & x_3=0\\ \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2}(d_{j_1 1}d_{j_2 1}d_{j_3 2}+d_{j_1 1}d_{j_2 2}d_{j_3 1}+d_{j_1 2}d_{j_2 1}d_{j_3 1})\alpha & x_3=1\\ \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2}(d_{j_1 1}d_{j_2 2}d_{j_3 2}+d_{j_1 2}d_{j_2 1}d_{j_3 2}+d_{j_1 2}d_{j_2 2}d_{j_3 1})\alpha & x_3=2\\ \sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} d_{j_1 2}d_{j_2 2}d_{j_3 2}\alpha & x_3=3\end{cases} \tag{42}$$

where

$$\alpha=\sum_{j_4=1}^{2}\sum_{j_5=1}^{2}\sum_{j_6=1}^{2}\pi_{2j_4}a_{j_4j_5}a_{j_5j_6}b_{j_6j_1}c_{j_1j_2}c_{j_2j_3}$$

Statistical Characteristics

$$\mu_r^1=\theta_1+2^r\theta_2+3^r\theta_3\ ,\forall r=1,2,3,4 \tag{43}$$

$$where,\ \theta_1=\sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2}(d_{j_1 1}d_{j_2 1}d_{j_3 2}+d_{j_1 1}d_{j_2 2}d_{j_3 1}+d_{j_1 2}d_{j_2 1}d_{j_3 1})\alpha \tag{44}$$

$$\theta_2=\sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2}(d_{j_1 1}d_{j_2 2}d_{j_3 2}+d_{j_1 2}d_{j_2 1}d_{j_3 2}+d_{j_1 2}d_{j_2 2}d_{j_3 1})\alpha \tag{45}$$

$$\theta_3=\sum_{j_1=1}^{2}\sum_{j_2=1}^{2}\sum_{j_3=1}^{2} d_{j_1 2}d_{j_2 2}d_{j_3 2}\alpha \tag{46}$$

$$\mu_2=\theta_1-\theta_1^2+4\theta_2-4\theta_1\theta_2-4\theta_2^2+9\theta_3-6\theta_1\theta_3-12\theta_2\theta_3-9\theta_3^2 \tag{47}$$

$$\begin{aligned}\mu_3=&\,2\theta_1^3+16\theta_2^3+24\theta_2^2(-1+3\theta_3)+3\theta_1^2(-1+4\theta_2+6\theta_3)+27\theta_3(1-3\theta_3+2\theta_3^2)\\&+2\theta_2(4-45\theta_3+54\theta_3^2)+\theta_1(1+24\theta_2^2-36\theta_3+54\theta_3^2+18\theta_2(-1+4\theta_3))\end{aligned} \tag{48}$$

$$\begin{aligned}\mu_4=&-3\theta_1^4-48\theta_2^4-96\theta_2^3(-1+3\theta_3)-6\theta_1^3(-1+4\theta_2+6\theta_3)-8\theta_2^2(8-63\theta_3+81\theta_3^2)\\&-81\theta_3(-1+4\theta_3-6\theta_3^2+3\theta_3^3)-8\theta_2(-2+39\theta_3-108\theta_3^2+81\theta_3^3)-2\theta_1^2(2+36\theta_2^2-45\theta_3+81\theta_3^2\\&+12\theta_2(-2+9\theta_3))-\theta_1(-1+96\theta_2^3+120\theta_3-378\theta_3^2+324\theta_3^3+24\theta_2^2(-5+18\theta_3)+8\theta_2(5-54\theta_3+8\end{aligned} \tag{49}$$

4 Numerical illustration

For the data analysis, we worked on secondary data on Diabetes used by [17]. The data set consists of 17 variables and 952 instances. For our proposed model, we took three variables: hidden layer one is Physical activity, hidden layer two is BMI and emission state is having the issue of Diabetes. From the data set, hidden layer one is the physical activity. We have four options: none, less than half an hour, more than half an hour and one hr or more. We classified these options into two categories: (i) 'NO' means; none and less than half an hr, and (ii) 'YES' means more than half an hr and one hr or more. For the second layer, we took BMI as obesity. If the BMI range is (i) less than 30, consider it as 'ABSENCE' and (ii) if it is more than 30, consider it as ‘PRESENCE’. For emission states, we consider having or not having diabetes. If it is not having, we consider it non-diabetes (the state is NO), whereas if it is having, we consider it Diabetes (the state is YES). The sample template of the Excel data [5] and graphical representation [6] show here.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	Age	Gender	Family_Diabetes	highBP	PhysicallyActive	BMI	Smoking	Alcohol	Sleep	SoundSleep	RegularMedicine	JunkFood	Stress	BPLevel	Pregancie	Pdiabetes	UriationFr	Diabeti
2	50-59	Male	no	yes	one hr or more	39	no	no	8	6	no	occasiona	sometimes	high	0	0	not much	no
3	50-59	Male	no	yes	less than half an hr	28	no	no	8	6	yes	very ofter	sometimes	normal	0	0	not much	no
4	40-49	Male	no	no	one hr or more	24	no	no	6	6	no	occasiona	sometimes	normal	0	0	not much	no
5	50-59	Male	no	no	one hr or more	23	no	no	8	6	no	occasiona	sometimes	normal	0	0	not much	no
6	40-49	Male	no	no	less than half an hr	27	no	no	8	8	no	occasiona	sometimes	normal	0	0	not much	no
7	40-49	Male	no	yes	none	21	no	yes	10	10	no	occasiona	sometimes	high	0	0	not much	yes
8	less than 4	Male	no	no	one hr or more	24	no	no	8	8	no	occasiona	sometimes	normal	0	0	not much	no
9	less than 4	Male	no	no	less than half an hr	20	no	no	7	7	yes	occasiona	sometimes	low	0	0	not much	no
10	40-49	Male	yes	no	one hr or more	23	no	no	7	7	no	occasiona	sometimes	normal	0	0	not much	no

Fig. 5 Sample data from excel sheet highlighted variables are studied in this paper

The scrutiny and data cleaning is considered for identifying the missing data. It is observed that out of 952 instances, only 948 cases are having the data presence instances. The data analysis has carried out on three platforms: Mathematica, Excel and R. After performing, values are followed for different layers and emission states.

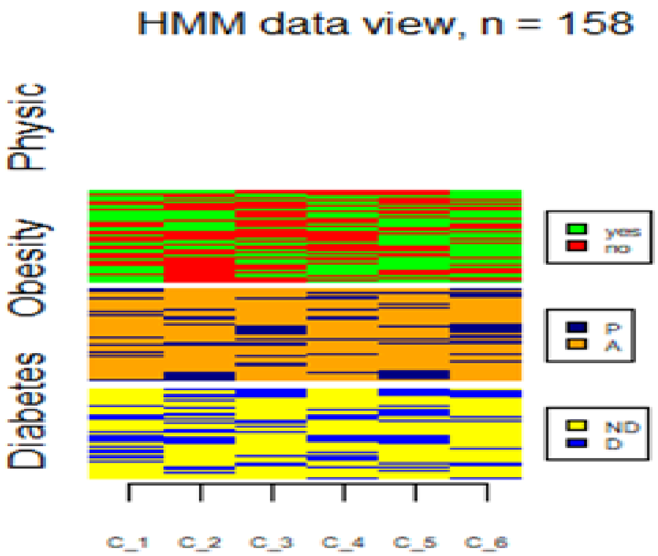

Fig. 6 Data visualization by seqHMM from R

4.1 Initial probability vector

Initial probabilities are starting probabilities from the hidden layer 1, calculated by how many 'NO' and 'YES' are in physical activity divided total number of instances. Frequency and initial probability vectors are given [4.1]. There are 467 no and 481 yes by using the formula of an initial probability vector given below.

$$\pi = [0.4926, .5074]$$

4.2 Intra-Layer Transition Probability Matrices (A & C)

The proposed model has two transition probabilities, physical activity and obesity they are given in [4.2.1, 4.2.2].

4.2.1 Layer-1

Transition probabilities follow the Markov chain property. Here TPM is defined and explained for the physical activity layer.

$$A = \begin{array}{cc} & \begin{array}{cc} No & Yes \end{array} \\ \begin{array}{c} No \\ Yes \end{array} & \begin{bmatrix} 0.5225 & 0.4775 \\ 0.4646 & 0.5354 \end{bmatrix} \end{array}$$

From the [4.2.1] state-1(No)it is observed that 52 % of the adults are not doing physical activity and around 48 % of adults later on are doing physical activity. From the state-2 (Yes) 46 % of adults are stopped physical activity and 54 % they still doing physical activity.

4.2.2 Layer-2

The TPM is defined [4.2.2] and explained for the obesity layer.

$$B = \begin{array}{c c} & \begin{array}{cc} Absent & Present \end{array} \\ \begin{array}{c} Absent \\ Present \end{array} & \begin{bmatrix} 0.8912 & 0.1088 \\ 0.3774 & 0.6226 \end{bmatrix} \end{array}$$

From the [4.2.2] State-1(Absent)it is observed that 89 % of the adults are not having obesity, but in the future, 11 % are about to be affected may be due to improper diet or physical activity. From state-2(Present) 38 % of adults came to normal from obesity by doing physical activity or proper diet management and 62 % still are having obesity.

4.3 Inter-Layer Transition (Connecting) Probability Matrix (B)

The connecting probabilities given in [4.3] are the probabilities between physical activity and obesity.

$$C = \begin{array}{c c} & \begin{array}{cc} Absent & Present \end{array} \\ \begin{array}{c} No \\ Yes \end{array} & \begin{bmatrix} 0.7516 & 0.2484 \\ 0.7983 & 0.2017 \end{bmatrix} \end{array}$$

From the [4.3] it is observed that those who are doing physical activity their an obesity percentage less than those who are doing. So this study recommended for adults, physical activity is important.

4.4 Emission Probabilities

The emission probabilities [4.4] are the observable probabilities between the latest hidden layer and the observable layer they follow.

$$D = \begin{array}{c c} & \begin{array}{cc} Non-Diabetes & Diabetes \end{array} \\ \begin{array}{c} Absent \\ Present \end{array} & \begin{bmatrix} 0.7442 & 0.2558 \\ 0.6385 & 0.3615 \end{bmatrix} \end{array}$$

From the [4.4] emission matrix observe that those who are having obesity they have a high chance of diabetes compared to non-obesity adults.

4.5 Schematic Diagram with Numeric

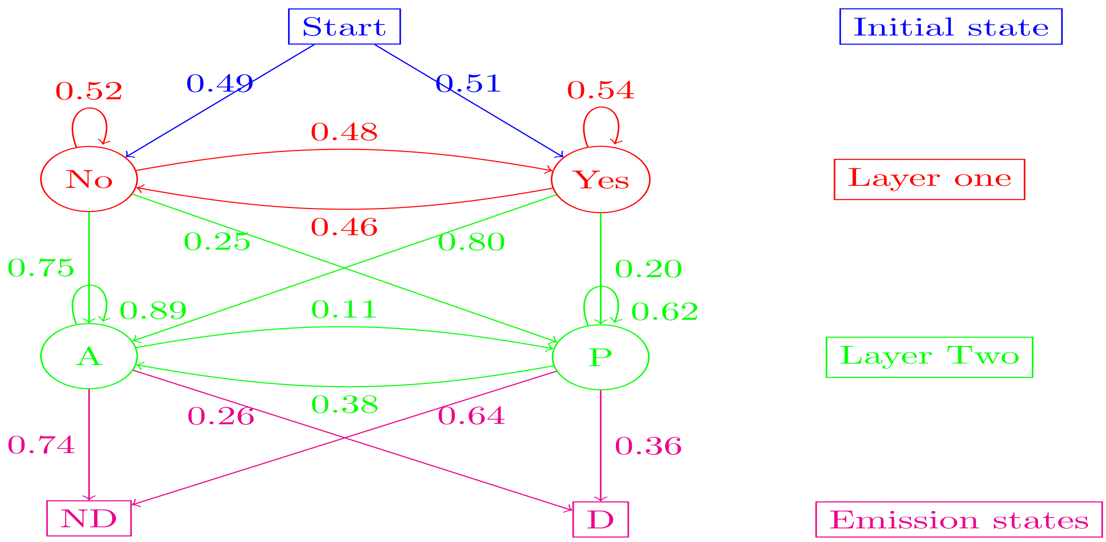

Fig. 7 Numerical schematic diagram of two-layered HMM

4.6 Probability Mass Function for Non-Diabetes

From the mathematical derivation section [3] probability mass functions with different state sequence lengths are followed.

X_{ND}	0	1	2	3
$P(x_1)$(one-state)	0.2795513	0.7204487	-	-
$P(x_3)$(two-state)	0.07913679	0.4007851	0.5200781	-
$P(x_5)$(three-state)	0.02256991	0.1691853	0.4324583	0.3757864

Table 1 Probability mass function for non-diabetes with different state sequence length

Table [1] observed that the probability of non-happening is low compared to the happening events but it shows a significant probability for diabetes. In the two-state and three-state probability of non-happening is low compared to happening.

4.7 Probability Mass Function for Diabetes

X_D	0	1	2	3
$P(x_1)$	0.7204487	0.2795513	-	-
$P(x_2)$	0.5200781	0.4007851	0.07913679	-
$P(x_3)$	0.3757864	0.4324583	0.1691853	0.02256991

Table 2 Probability mass function for diabetes

Table [2] observed that the probability of happening is low compared to the non-happening events but shows a significant probability for diabetes. In the two-state and three-state probability of happening is significant for the diabetes state.

4.8 Statistical Characteristics for Non-Diabetes

From the pmf from [4.6] statistical characteristics was calculated and verified with mathematical expressions.

Sno	Statistical Measure	One-State	Two-States	Three-States
1	Mean	0.7204487	1.440941	2.161461
2	Variance	0.2014024	0.4047856	0.6091817
3	μ_3	-0.08879779	-0.1802507	-0.2732452
4	μ_4	0.07971363	0.4071191	0.9865976
5	Skewness	0.9651853	0.4898675	0.3302666
6	γ_1	0.9824385	0.6999053	0.5746883
7	Kurtosis	1.965185	2.484685	2.65856
8	γ_2	-1.034815	-0.5153154	-0.3414403

Table 3 Statistical characteristics for non-diabetes

Table [3] observes that when state sequence increases, variance also increases, the third central moment has a negative trend and the fourth central moment is positive. In the shaping parameter, probability distribution has positive Skewness for all three states and Kurtosis have higher peakedness.

4.9 Statistical Characteristics for Diabetes

The statistical characteristics for diabetes were obtained from [4.7] and verified with mathematical expressions [3].

Sno	Statistical Measure	One-State	Two-States	Three-States
1	Mean	0.2795513	0.5590587	0.8385387
2	Variance	0.2014024	0.4047856	0.6091817
3	μ_3	0.08879779	0.1802507	0.2732452
4	μ_4	0.07971363	0.4071191	0.9865976
5	Skewness	0.9651853	0.4898675	0.3302666
6	γ_1	0.9824385	0.6999053	0.5746883
7	Kurtosis	1.965185	2.484685	2.65856
8	γ_2	-1.034815	-0.5153154	-0.3414403

Table 4 Statistical characteristics for diabetes

Table [4] shows that the diabetes state average is in increasing when the state sequence increases and the remaining statistical characteristics except μ_3 are the same as non-diabetes.

5 Results and Discussions

For the proposed Layer Hidden Markov Model (LHMM), the results are as follows, formulated in mathematical form for obtaining the finding probability of emission state sequence by the forward algorithm shown in section [3]. Further based on happening or occurrences derived, the probability mass function for emission state sequence for non-diabetes and diabetes with statistical characteristics derivations verified with Mathematica and MS-Excel by numerical examples in sections [3.2 3.3 3.4]. For the numerical illustrations, worked on secondary data collected by a researcher. Taken three variables from the data: physical activity, BMI and diabetes. Here assumptions for the variables are as follows physical activity is hidden layer 1, BMI is hidden layer 2 influenced by physical activity and Diabetes is influenced by BMI. Performed numerical illustration in RStudio with manual coding. The statistical results are as follows: In one state sequence, the non-diabetes probability is 0.7204 and Diabetes is 0.2796. Similarly for 2 emission states like P(diabetes,non-diabetes)= 0.200417 , P(non-diabetes,diabetes)= 0.2003681 , P(non-diabetes non-diabetes)=0.5200781 and P(diabetes diabetes)= 0.07913679. For three states P(non-diabetes non-diabetes non-diabetes)= 0.3757864 ,P(diabetes diabetes diabetes)= 0.02256991,P(diabetes non-diabetes non-diabetes)= 0.1443435, P(non-diabetes diabetes non-diabetes)= 0.1438234, P(non-diabetes non-diabetes diabetes)= 0.1442915, P(non-diabetes diabetes diabetes)= 0.05654473, P(diabetes non-diabetes diabetes)= 0.05607364 and P(diabetes diabetes non-diabetes)= 0.05656694. when the state sequence increase then the mean and variance also increase. In both cases Skewness is positive, and Kurtosis is high peakedness.

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